

## APPLICATION OF DERIVATIVES

95. **Statements-1:** For the circle  $(x - 1)^2 + (y - 1)^2 = 1$ , the tangent at the point  $(1, 0)$  is the x-axis.  
**Statements-2:** the derivative of a single valued function  $y = f(x)$  at  $x = a$  is the slope of the tangent drawn to the curve at  $x = a$ .
96. **Statements-1:** Both  $\sin x$ , and  $\cos x$  are decreasing functions in  $\left(\frac{\pi}{2}, \pi\right)$  [ Good ]  
**Statements-2:** If a differentiable function decreases in an interval  $(a, b)$  then its derivative also decreases in  $(a, b)$ .
97. **Statements-1:**  $e^\pi > \pi^e$  [ Good ]  
**Statements-2:** The function  $x^{\frac{1}{x}}$  ( $x > 0$ ) has a local maximum at  $x = e$
98. **Statements-1:** Conditions of LMVT fail in  $f(x) = |x - 1|$  ( $x - 1$ )  
**Statements-2:**  $|x - 1|$  is not differentiable at  $x = 1$
99. Let  $f(x) = \sum_{i=1}^n (x - x_i)^2$   
**Statement-1 :** Minimum value of  $f(x)$  occurs at  $x = \frac{\sum x_i}{n}$   
**Statement-2 :** Minimum of  $f(x) = ax^2 + bx + c$  ( $a > 0$ ) occurs at  $x = -b/2a$ .
100. **Statement-1 :**  $\alpha^\beta > \beta^\alpha$ , for  $2.91 < \alpha < \beta$   
**Statement-2 :**  $f(x) = \frac{\log_e x}{x}$  is a decreasing function for  $x > e$ .
101. **Statement-1 :** Total number of critical points of  $f(x) = \max. \{1/2, \sin x, \cos x\} - \pi \leq x \leq \pi$  are 5  
**Statement-2 :** Total number of critical points of  $f(x) = \max \{1/2, x, \cos x\} - \pi \leq x \leq \pi$  are 2
102. Let  $f(x) = 5p^2 + 4(x - 1) - x^2$ ,  $x \in \mathbb{R}$  and  $p$  is a real constant  
**Statement-1 :** If the maximum values of  $f(x)$  is 20, then  $p = -2$ .  
**Statement-2 :** If the maximum value of  $f(x)$  is 20, then  $p = 2$ .
103. Let  $f(x) = \sin^{-1} x + \cos^{-1} x + \tan^{-1} x$  and  $x \in [-1, 1]$   
**Statements-1:** Range of  $f(x)$  is  $\left[\frac{\pi}{4}, \frac{3\pi}{4}\right]$ .  
**Statements-2:**  $f(x)$  is an increasing function.
104. Let  $f(x) = x^3$   
**Statements-1:**  $x = 0$ , in the point of inflexion for  $f(x)$   
**Statements-2:**  $f''(x) < 0$  for  $x < 0$  and  $f''(x) > 0$  for  $x > 0$ .
105. Suppose  $f(x) = \frac{x^2}{2} + \ln x + 2 \cos x$   
**Statements-1:**  $f$  is an increasing function.  
**Statements-2:** derivative of  $f(x)$  with respect to  $x$  is always greater than zero.
106. Let  $0 < x \leq \frac{\pi}{2}$  and  $f(x) = \frac{\sin x}{x}$   
**Statements-1:** The minimum value of  $f$  is  $\frac{2}{\pi}$ , attained at  $x = \frac{\pi}{2}$ .  
**Statements-2:**  $0 < \sin x < x$ ,  $\forall x \in \left(0, \frac{\pi}{2}\right]$ .
107. **Statements-1:** The equation  $x^2 = x \sin x + \cos x$  has only one solution.  
**Statements-2:** The derivative of the function  $x^2 - x \sin x - \cos x$  is  $x(2 - \cos x)$ .
108. **Statement-1 :** Angle of intersects in between  $y = x^2$  and  $6y = 7 - x^3$  at  $(1, 1)$  is  $\pi/4$   
**Statement-2 :** Angle of intersection between any two curve is angle between the tangents at the point of intersection.
109. **Statement - 1 :** The curve  $y = x^{1/3}$  has a point of inflection at  $x = 0$   
**Statement - 2 :** A point where  $y''$  fails to exist can be a point of inflection

110. Let  $f(x)$  and  $g(x)$  are two positive and increasing function  
**Statement – 1 :** If  $(f(x)g(x))$  is decreasing then  $f(x) < 1$   
**Statement – 2 :** If  $f(x)$  is decreasing then  $f'(x) < 0$  and increasing then  $f'(x) > 0$  for all  $x$ .
111. **Statement – 1 :** If  $f(0) = 0$ ,  $f'(x) = \ln(x + \sqrt{1+x^2})$ , then  $f(x)$  is positive for all  $x \in \mathbb{R}_0$   
**Statements-2:**  $f(x)$  is increasing for  $x > 0$  and decreasing for  $x < 0$ .
112. **Statements-1:** The two curves  $y^2 = 4x$  and  $x^2 + y^2 - 6x + 1 = 0$  at the point  $(1, 2)$  intersect orthogonally.  
**Statements-2:** Two curves  $y = f(x)$  &  $y = g(x)$  intersect orthogonally at  $(x_1, y_1)$  if  $(f'(x_1) \cdot g'(x_1)) = -1$ .
113. **Statements-1:** If  $27a + 9b + 3c + d = 0$ , then the equation  $4ax^3 + 3bx^2 + 2cx + d = 0$  has atleast one real root lying between  $(0, 3)$   
**Statements-2:** If  $f(x)$  is continuous in  $[a, b]$ , derivable in  $(a, b)$ , then at least one point  $c \in (a, b)$  such that  $f'(c) = 0$ .
114. **Statements-1:**  $f(x) = \{x\}$  has local minima at  $x = 1$ .  
**Statements-2:**  $x = a$  will be local minima for  $y = f(x)$  provided  $\lim_{x \rightarrow a^-} f(x) > f(a)$  also  
 $\lim_{x \rightarrow a^+} f(x) > f(a)$ .
115. **Statements-1:**  $f(x) = \frac{1}{2} - x$ ;  $x < \frac{1}{2}$   
 $= \left(\frac{1}{2} - x\right)^2$ ;  $x \geq \frac{1}{2}$ . Mean value theorem is applicable in the interval  $[0, 1]$ .  
**S-2:** For application of mean value theorem,  $f(x)$  must be continuous in  $[0, 1]$  and differentiable in  $(0, 1)$ .
116. **Statements-1:** For some  $0 < x_1 < x_2 < \pi/2$ ,  $\tan^{-1}x_2 - \tan^{-1}x_1 < x_2 - x_1$   
**Statements-2:** If  $f(x) > f(x_1) \Rightarrow x_2 > x_1$   
 function is always increasing
117. **Statements-1:** The graph of a continuous function  $y = f(x)$  has a cusp at point  $x = c$  if  $f''(x)$  has same sign on both sides of  $c$ .  
**Statements-2:** The concavity at any point  $x = c$  depends upon  $f''(x)$ . If  $f''(x) < 0$  or  $f''(x) > 0$  the function is either concave up or concave down.
118. **Statements-1:** If  $f$  be a function defined for all  $x$  such that  $|f(x) - f(y)| < (x - y)^2$  then  $f$  is constant  
**Statements-2:** If  $\alpha(x) < \beta(x) < \gamma(x)$  for all  $x$  and  $\lim_{x \rightarrow a} \alpha(x) = \lim_{x \rightarrow a} \gamma(x) = L \Rightarrow \lim_{x \rightarrow a} \beta(x) = L$
119. **Statements-1:**  $f: \mathbb{R} \rightarrow \mathbb{R}$  be a function such that  $f(x) = x^3 + x^2 + 3x + \sin x$ . Then  $f$  is one-one.  
**Statements-2:**  $f(x)$  is neither increasing nor decreasing.
120. **Statements-1:** If  $\alpha$  &  $\beta$  are any two roots of equation  $e^x \cos x = 1$ , then the equation  $e^x \sin x - 1 = 0$  has at least one root in  $(\alpha, \beta)$   
**Statements-2:**  $f$  is continuous in  $[\alpha, \beta]$ .  $f$  is derivable in  $(\alpha, \beta)$ .  $f(\alpha) = f(\beta)$  then there exists  $x \in (\alpha, \beta)$  such that  $f'(x) = 0$
121. **Statements-1:** The minimum value of the expression  $x^2 + 2bx + c$  is  $c - b^2$ .  
**Statements-2:** The first order derivative of the expression at  $x = -b$  is zero and second derivative is always positive.
122. **Statements-1:** Let  $\phi(x) = \sin(\cos x)$  in  $\left[0, \frac{\pi}{2}\right]$  then  $\phi(x)$  is decreasing in  $\left[0, \frac{\pi}{2}\right]$   
**Statements-2:**  $\phi'(x) \leq 0 \forall x \in \left[0, \frac{\pi}{2}\right]$
123. **Statements-1:** The function  $f(x) = x^4 - 8x^3 + 22x^2 - 24x + 21$  is decreasing for every  $x \in (2, 3) \cup (-\infty, 1)$   
**Statements-2:**  $f'(x) > 0$  for the given values of  $x$ .
124. **Statements-1:** For the function  $f(x) = x^x$ ,  $x = 1/e$  is a point of local minimum.  
**Statements-2:**  $f'(x)$  changes its sign from -ve to positive in neighbourhood of  $x = 1/e$ .
125. **Statements-1:** Consider the function  $f(x) = (x^3 - 6x^2 + 12x - 8)e^x$  is neither maximum nor minimum let  $x = 2$   
**Statements-2:**  $f'(x) = 0$ ,  $f''(x) = 0$ ,  $f'''(x) \neq 0$  at  $x = 2$
126. **Statements-1:** Consider the function  $f(x) \frac{f(x_1 + x_2)}{2} < \frac{f(x_1) + f(x_2)}{2}$   
**Statements-2:**  $f'(x) > 0$ ,  $f''(x) > 0$  where  $x_1 < x_2$
127. Consider the following function with regard to the function

$$f(x) = (x^3 - 6x^2 + 12x - 8) e^x$$

**Statement-1:**  $f(x)$  is neither maximum nor minimum at  $x = 2$

**Statement-2:**  $f'(x) = 0$ ,  $f''(x) = 0$ ,  $f'''(x) \neq 0$  at  $x = 2$ .

128. **Statements-1:** Equation  $f(x) = x^3 + 9x^2 + 2ax + a^2 + a + 1 = 0$  has at least one real negative root.  
**Statements-2:** Every equation of odd degree has at least one real root whose sign is opposite to that of its constant term.

**ANSWER**

- |        |        |        |        |        |        |        |
|--------|--------|--------|--------|--------|--------|--------|
| 95. B  | 96. C  | 97. A  | 98. D  | 99. A  | 100. A | 101. A |
| 102. A | 103. A | 104. A | 105. A | 106. B | 107. D | 108. D |
| 109. A | 110. A | 111. A | 112. D | 113. A | 114. A | 115. D |
| 116. A | 117. A | 118. A | 119. C | 120. A | 121. A | 122. A |
| 123. C | 124. A | 125. A | 126. A | 127. A | 128. A |        |

## Que. from Compt. Exams

- $\frac{d}{dx} \tan^{-1} \left[ \frac{\cos x - \sin x}{\cos x + \sin x} \right] =$  [AISSE 1985, 87; DSSE 1982,84; MNR 1985; Karnataka CET 2002; RPET 2002, 03]
 

(a)  $\frac{1}{2(1+x^2)}$  (b)  $\frac{1}{1+x^2}$  (c) 1 (d) -1
- If  $y = \frac{x}{2} \sqrt{a^2 + x^2} + \frac{a^2}{2} \log(x + \sqrt{x^2 + a^2})$ , then  $\frac{dy}{dx} =$  [AISSE 1983]
 

(a)  $\sqrt{x^2 + a^2}$  (b)  $\frac{1}{\sqrt{x^2 + a^2}}$  (c)  $2\sqrt{x^2 + a^2}$  (d)  $\frac{2}{\sqrt{x^2 + a^2}}$
- If  $y = \cot^{-1}(\cos 2x)^{1/2}$ , then the value of  $\frac{dy}{dx}$  at  $x = \frac{\pi}{6}$  will be [IIT 1992]
 

(a)  $\left(\frac{2}{3}\right)^{1/2}$  (b)  $\left(\frac{1}{3}\right)^{1/2}$  (c)  $(3)^{1/2}$  (d)  $(6)^{1/2}$
- If  $f(x+y) = f(x).f(y)$  for all  $x$  and  $y$  and  $f(5) = 2$ ,  $f(0) = 3$ , then  $f'(5)$  will be [IIT 1981; Karnataka CET 2000; UPSEAT 2002; MP PET 2002; AIEEE 2002]
 

(a) 2 (b) 4 (c) 6 (d) 8
- If  $xe^{-xy} = y + \sin^2 x$ , then at  $x = 0$ ,  $\frac{dy}{dx} =$  [IIT 1996]
 

(a) -1 (b) -2 (c) 1 (d) 2
- If  $u(x,y) = y \log x + x \log y$ , then  $u_x u_y - u_x \log x - u_y \log y + \log x \log y =$  [EAMCET 2003]
 

(a) 0 (b) -1 (c) 1 (d) 2
- If  $y = f\left(\frac{2x-1}{x^2+1}\right)$  and  $f'(x) = \sin x^2$ , then  $\frac{dy}{dx} =$  [IIT 1982]
 

(a)  $\frac{6x^2 - 2x + 2}{(x^2 + 1)^2} \sin\left(\frac{2x-1}{x^2+1}\right)^2$  (b)  $\frac{6x^2 - 2x + 2}{(x^2 + 1)^2} \sin^2\left(\frac{2x-1}{x^2+1}\right)$   
 (c)  $\frac{-2x^2 + 2x + 2}{(x^2 + 1)^2} \sin^2\left(\frac{2x-1}{x^2+1}\right)$  (d)  $\frac{-2x^2 + 2x + 2}{(x^2 + 1)^2} \sin\left(\frac{2x-1}{x^2+1}\right)^2$
- If  $x = \sec \theta - \cos \theta$  and  $y = \sec^n \theta - \cos^n \theta$ , then [IIT 1989]
 

(a)  $(x^2 + 4) \left(\frac{dy}{dx}\right)^2 = n^2(y^2 + 4)$  (b)  $(x^2 + 4) \left(\frac{dy}{dx}\right)^2 = x^2(y^2 + 4)$   
 (c)  $(x^2 + 4) \left(\frac{dy}{dx}\right)^2 = (y^2 + 4)$  (d) None of these
- If  $y = x^{x^{\dots \infty}}$ , then  $\frac{dy}{dx} =$  [UPSEAT 2004; DCE 2000]

(a)  $\frac{y^2}{x(1+y \log x)}$  (b)  $\frac{y^2}{x(1-y \log x)}$  (c)  $\frac{y}{x(1+y \log x)}$  (d)  $\frac{y}{x(1-y \log x)}$

10. If  $y = (x \log x)^{\log \log x}$ , then  $\frac{dy}{dx} =$  [Roorkee 1981]

(a)  $(x \log x)^{\log \log x} \left\{ \frac{1}{x \log x} (\log x + \log \log x) + (\log \log x) \left( \frac{1}{x} + \frac{1}{x \log x} \right) \right\}$   
 (b)  $(x \log x)^{x \log x} \log \log x \left[ \frac{2}{\log x} + \frac{1}{x} \right]$  (c)  $(x \log x)^{x \log x} \frac{\log \log x}{x} \left[ \frac{1}{\log x} + 1 \right]$

(d) None of these

11.  $\frac{d}{dx} \left[ \tan^{-1} \frac{\sqrt{1+x^2} + \sqrt{1-x^2}}{\sqrt{1+x^2} - \sqrt{1-x^2}} \right] =$  [Roorkee 1980; Karnataka CET 2005]

(a)  $\frac{-x}{\sqrt{1-x^4}}$  (b)  $\frac{x}{\sqrt{1-x^4}}$  (c)  $\frac{-1}{2\sqrt{1-x^4}}$  (d)  $\frac{1}{2\sqrt{1-x^4}}$

12. If  $\sqrt{1-x^6} + \sqrt{1-y^6} = a^3(x^3 - y^3)$ , then  $\frac{dy}{dx} =$  [Roorkee 1994]

(a)  $\frac{x^2}{y^2} \sqrt{\frac{1-x^6}{1-y^6}}$  (b)  $\frac{y^2}{x^2} \sqrt{\frac{1-y^6}{1-x^6}}$  (c)  $\frac{x^2}{y^2} \sqrt{\frac{1-y^6}{1-x^6}}$  (d) None of these

13. If  $y = \sec^{-1} \frac{2x}{1+x^2} + \sin^{-1} \frac{x-1}{x+1}$ , then  $\frac{dy}{dx}$  is equal to [Pb. CET 2000]

(a) 1 (b)  $\frac{x-1}{x+1}$  (c) Does not exist (d) None of these

14. The derivative of  $\tan^{-1} \left( \frac{\sqrt{1+x^2}-1}{x} \right)$  with respect to  $\tan^{-1} \left( \frac{2x\sqrt{1-x^2}}{1-2x^2} \right)$  at  $x=0$ , is

(a)  $\frac{1}{8}$  (b)  $\frac{1}{4}$  (c)  $\frac{1}{2}$  (d) 1

15. If  $y^2 = p(x)$  is a polynomial of degree three, then  $2 \frac{d}{dx} \left\{ y^3 \cdot \frac{d^2 y}{dx^2} \right\} =$  [IIT 1988; RPET 2000]

(a)  $p'''(x) + p'(x)$  (b)  $p''(x) \cdot p'''(x)$  (c)  $p(x) \cdot p'''(x)$  (d) Constant

16. Let  $f(x)$  and  $g(x)$  be two functions having finite non-zero 3<sup>rd</sup> order derivatives  $f'''(x)$  and  $g'''(x)$  for all  $x \in R$ . If  $f(x)g(x) = 1$  for all  $x \in R$ , then  $\frac{f'''}{f'} - \frac{g'''}{g'}$  is equal to

(a)  $3 \left( \frac{f''}{g} - \frac{g''}{f} \right)$  (b)  $3 \left( \frac{f''}{f} - \frac{g''}{g} \right)$  (c)  $3 \left( \frac{g''}{g} - \frac{f''}{f} \right)$  (d)  $3 \left( \frac{f''}{f} - \frac{g''}{f} \right)$

17. If  $I_n = \frac{d^n}{dx^n} (x^n \log x)$ , then  $I_n - nI_{n-1} =$  [EAMCET 2003]

(a)  $n$  (b)  $n-1$  (c)  $n!$  (d)  $(n-1)!$

18. If  $x = \sin t$  and  $y = \sin pt$ , then the value of  $(1-x^2) \frac{d^2 y}{dx^2} - x \frac{dy}{dx} + p^2 y$  is equal to [Pb. CET 2002]

(a) 0 (b) 1 (c) -1 (d)  $\sqrt{2}$

19. Let  $f: (0, +\infty) \rightarrow R$  and  $F(x) = \int_0^x f(t) dt$ . If  $F(x^2) = x^2(1+x)$ , then  $f(4)$  equals [IIT Screening 2001]

(a)  $\frac{5}{4}$  (b) 7 (c) 4 (d) 2

20. The volume of a spherical balloon is increasing at the rate of 40 cubic centimetre per minute. The rate of change of the surface of the balloon at the instant when its radius is 8 centimetre, is [Roorkee 1983]

(a)  $\frac{5}{2}$  sq cm/min (b) 5 sq cm/min (c) 10 sq cm/min (d) 20 sq cm/min

21. A man of height 1.8 metre is moving away from a lamp post at the rate of 1.2 m/sec. If the height of the lamp post be 4.5 metre, then the rate at which the shadow of the man is lengthening is [MP PET 1989]

(a) 0.4 m/sec (b) 0.8 m/sec (c) 1.2 m/sec (d) None of these

22. The radius of the cylinder of maximum volume, which can be inscribed in a sphere of radius  $R$  is [AMU 1999]  
 (a)  $\frac{2}{3}R$  (b)  $\sqrt{\frac{2}{3}}R$  (c)  $\frac{3}{4}R$  (d)  $\sqrt{\frac{3}{4}}R$
23. The distance travelled  $s$  (in metre) by a particle in  $t$  seconds is given by,  $s = t^3 + 2t^2 + t$ . The speed of the particle after 1 second will be [UPSEAT 2003]  
 (a) 8 cm/sec (b) 6 cm/sec (c) 2 cm/sec (d) None of these
24. If  $y = 4x - 5$  is tangent to the curve  $y^2 = px^3 + q$  at  $(2, 3)$ , then [IIT 1994; UPSEAT 2001]  
 (a)  $p = 2, q = -7$  (b)  $p = -2, q = 7$  (c)  $p = -2, q = -7$  (d)  $p = 2, q = 7$
25. At what points of the curve  $y = \frac{2}{3}x^3 + \frac{1}{2}x^2$ , tangent makes the equal angle with axis [UPSEAT 1999]  
 (a)  $\left(\frac{1}{2}, \frac{5}{24}\right)$  and  $\left(-1, -\frac{1}{6}\right)$  (b)  $\left(\frac{1}{2}, \frac{4}{9}\right)$  and  $(-1, 0)$  (c)  $\left(\frac{1}{3}, \frac{1}{7}\right)$  and  $\left(-3, \frac{1}{2}\right)$  (d)  $\left(\frac{1}{3}, \frac{4}{47}\right)$  and  $\left(-1, -\frac{1}{3}\right)$
26. If the normal to the curve  $y = f(x)$  at the point  $(3, 4)$  makes an angle  $\frac{3\pi}{4}$  with the positive  $x$ -axis then  $f'(3)$  is equal to  
 (a)  $-1$  (b)  $-\frac{3}{4}$  (c)  $\frac{4}{3}$  (d)  $1$
27. The point(s) on the curve  $y^3 + 3x^2 = 12y$  where the tangent is vertical (parallel to  $y$ -axis), is (are) [IIT Screening 2002]  
 (a)  $\left(\pm \frac{4}{\sqrt{3}}, -2\right)$  (b)  $\left(\pm \frac{\sqrt{11}}{3}, 1\right)$  (c)  $(0, 0)$  (d)  $\left(\pm \frac{4}{\sqrt{3}}, 2\right)$
28. Let  $f(x) = \int_0^x \frac{\cos t}{t} dt, x > 0$  then  $f(x)$  has [Kurukshetra CEE 2002]  
 (a) Maxima when  $n = -2, -4, -6, \dots$  (b) Maxima when  $n = -1, -3, -5, \dots$   
 (c) Minima when  $n = 0, 2, 4, \dots$  (d) Minima when  $n = 1, 3, 5, \dots$
29. If  $f(x) = x^2 + 2bx + 2c^2$  and  $g(x) = -x^2 - 2cx + b^2$  such that  $\min f(x) > \max g(x)$ , then the relation between  $b$  and  $c$  is [IIT Screening 2003]  
 (a) No real value of  $b$  and  $c$  (b)  $0 < c < b\sqrt{2}$   
 (c)  $|c| < |b|\sqrt{2}$  (d)  $|c| > |b|\sqrt{2}$
30.  $N$  characters of information are held on magnetic tape, in batches of  $x$  characters each; the batch processing time is  $\alpha + \beta x^2$  seconds;  $\alpha$  and  $\beta$  are constants. The optimal value of  $x$  for fast processing is [MNR 1986]  
 (a)  $\frac{\alpha}{\beta}$  (b)  $\frac{\beta}{\alpha}$  (c)  $\sqrt{\frac{\alpha}{\beta}}$  (d)  $\sqrt{\frac{\beta}{\alpha}}$
31. On the interval  $[0, 1]$ , the function  $x^{25}(1-x)^{75}$  takes its maximum value at the point [IIT 1995]  
 (a) 0 (b)  $1/2$  (c)  $1/3$  (d)  $1/4$
32. The function  $f(x) = \int_{-1}^x t(e^t - 1)(t-1)(t-2)^3(t-3)^5 dt$  has a local minimum at  $x =$  [IIT 1999]  
 (a) 0 (b) 1 (c) 2 (d) 3
33. The maximum value of  $\exp(2 + \sqrt{3} \cos x + \sin x)$  is [AMU 1999]  
 (a)  $\exp(2)$  (b)  $\exp(2 - \sqrt{3})$  (c)  $\exp(4)$  (d) 1
34. If the function  $f(x) = 2x^3 - 9ax^2 + 12a^2x + 1$ , where  $a > 0$  attains its maximum and minimum at  $p$  and  $q$  respectively such that  $p^2 = q$ , then  $a$  equals [AIEEE 2003]  
 (a) 3 (b) 1 (c) 2 (d)  $\frac{1}{2}$
35. The function  $f(x) = \frac{\ln(\pi + x)}{\ln(e + x)}$  is [IIT 1995]  
 (a) Increasing on  $[0, \infty)$  (b) Decreasing on  $[0, \infty)$

(c) Decreasing on  $\left[0, \frac{\pi}{e}\right)$  and increasing on  $\left[\frac{\pi}{e}, \infty\right)$

(d) Increasing on  $\left[0, \frac{\pi}{e}\right)$  and decreasing on  $\left[\frac{\pi}{e}, \infty\right)$

36. The function  $f(x) = \sin^4 x + \cos^4 x$  increases, if

[IIT 1999; Pb. CET 2001]

(a)  $0 < x < \frac{\pi}{8}$  (b)  $\frac{\pi}{4} < x < \frac{3\pi}{8}$

(c)  $\frac{3\pi}{8} < x < \frac{5\pi}{8}$  (d)  $\frac{5\pi}{8} < x < \frac{3\pi}{4}$

37. Let  $h(x) = f(x) - (f(x))^2 + (f(x))^3$  for every real number  $x$ . Then

[IIT 1998]

(a)  $h$  is increasing whenever  $f$  is increasing

(b)

$h$  is increasing whenever  $f$  is decreasing

(c)  $h$  is decreasing whenever  $f$  is decreasing

(d)

Nothing can be said in general

38. In  $[0, 1]$  Lagrange's mean value theorem is NOT applicable to

[IIT Screening 2003]

(a)  $f(x) = \begin{cases} \frac{1}{2} - x, & x < \frac{1}{2} \\ \left(\frac{1}{2} - x\right)^2, & x \geq \frac{1}{2} \end{cases}$  (b)  $f(x) = \begin{cases} \frac{\sin x}{x}, & x \neq 0 \\ 1, & x = 0 \end{cases}$

(c)  $f(x) = x|x|$

(d)  $f(x) = x|x|$

39. If the function  $f(x) = x^3 - 6ax^2 + 5x$  satisfies the conditions of Lagrange's mean value theorem for the interval  $[1, 2]$  and the tangent to the curve  $y = f(x)$  at  $x = \frac{7}{4}$  is parallel to the chord that joins the points of intersection of the curve with the ordinates  $x = 1$  and  $x = 2$ . Then the value of  $a$  is

[MP PET 1998]

(a)  $\frac{35}{16}$

(b)  $\frac{35}{48}$

(c)  $\frac{7}{16}$

(d)  $\frac{5}{16}$

40. Let  $f(x) = \begin{cases} x^\alpha \ln x, & x > 0 \\ 0, & x = 0 \end{cases}$ , Rolle's theorem is applicable to  $f$  for  $x \in [0, 1]$ , if  $\alpha =$

[IIT Screening 2004]

(a)  $-2$

(b)  $-1$

(c)  $0$

(d)  $\frac{1}{2}$

**Que. from Compt. Exams**

1	d	2	a	3	a	4	c	5	c
6	c	7	d	8	a	9	b	10	a
11	a	12	c	13	c	14	b	15	c
16	b	17	d	18	a	19	c	20	c
21	b	22	b	23	a	24	a	25	a
26	d	27	d	28	b,d	29	d	30	c
31	d	32	b,d	33	c	34	c	35	b
36	b	37	a, c	38	a	39	b	40	d

**For 39 Years Que. of IIT-JEE (Advanced)**  
**& 15 Years Que. of AIEEE (JEE Main)**  
**we have already distributed a book**